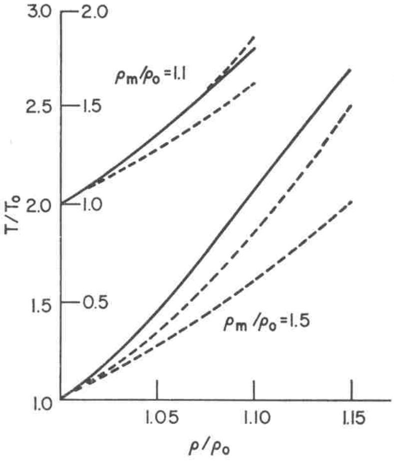

FIG. 2. Non-dimensionalized plots of temperature versus density in two shock waves. For one wave the maximum density corresponds to $\rho_m/\rho_0 = 1{\cdot}15$ and for the other $\rho_m/\rho_0 = 1{\cdot}10$. As in Fig. 1, the solid curves are computed from (26) with $\mu = 5$, $\beta T_0 - 10^{-2}$ and $\rho_h = \rho_0$ while the dotted curves are computed from (19) and (27).

There is a striking resemblance in the second term of (28) to the Murnaghan equation frequently employed to represent the variation of bulk modulus with density. Consider the usual definition of compressibility $\varkappa = -v\, dp/dv$ in which the specific volume v is the reciprocal of the density ρ. Differentiation of (28) shows that

$$\varkappa = (\rho K/\rho_0)\{\ln(\rho/\rho_0) + 1 + \beta T_0[(1+\mu)(\rho/\rho_0)^{\mu} - 1]\}. \tag{29}$$

The first major effect of accounting for temperature changes appears in (29) which shows that for very weak shocks, $\rho \to \rho_0$, the effective compressibility $\varkappa$ equals the isothermal bulk modulus K times $(1+\beta T_0 \mu)$, i.e. $\varkappa_0 = K^*$. Taking βT_0 as 0·01 and μ as 5 indicates an increase of the order of five percent of the adiabatic value above the isothermal compressibility. Although this difference increases with increasing compression it has no significant practical effects since investigators who make use of temperature-independent analyses ordinarily rely upon experimentally-determined adiabatic pressure–volume relations to find appropriate constants. For example, using a temperature-independent pressure-density relation in (KG), we showed a good fit to dynamic experimental data for iron with $K = 2000$ kb. To fit the same data using (28) with $\beta T_0 = 0{\cdot}01$ and $\mu = 5$, the value for K must be taken to be approximately 1800 kb. Thus, given experimental data can be adequately represented over a limited range by means of either type of equation.

The main purpose, therefore, in trying to calculate temperatures in the analysis of plane shock waves is not because of their effect on the pressure–density relation but mainly because of their influence on the plastic response of the material. This response is affected in two separate ways as will be illustrated in the numerical calculations following. First, the small differences in the pressure–density relation give rise to rather substantial differences in the deviatoric stress. The viscoplastic response of materials is ordinarily postulated as being strongly influenced by the stress deviator. Secondly, the plastic response of materials is also usually postulated as being strongly influenced directly by the temperature. It is these two effects that underly the importance of the foregoing theory by enabling the previous method of analysis to be meaningfully extended into the range of shock pressures where temperature increases are substantial, and by providing means to estimate the applicable limits of this range.

3. NUMERICAL ANALYSES

We are now in a position to compute the main features of a steady-state plastic wave taking temperature changes into account by using the simple method proposed in (KG). The following quantities have been expressed directly as functions of the density:

stress	(23),
temperature	(26),
pressure	(10),
total strain	(15),
deviatoric stress	(11),
plastic strain	(17).

In order to complete the description of the problem the material plastic-strain response must be specified. Equations (10), (11) and (26) and the plastic strain response comprise the material constitutive relations.

In (KG), heuristic arguments were presented to show that in an isothermal theory the plastic strain response ought to be given by some functional dependence of the plastic strain rate, $\dot{\varepsilon}^p$, on the plastic strain and the reduced deviatoric stress, $\rho_0(\sigma+\rho)/\rho$. Extension of these arguments to the present theory would show that this functional dependence might also include the temperature rise. Thus, we complete the problem description by specifying some plastic strain rate equation:

$$\dot{\varepsilon}^p = f(\varepsilon^p, \rho(\sigma+p)/\rho_0, \theta). \tag{30}$$

We point out that the restriction of the arguments of f to these three variables is dictated solely by taste. The numerical analysis depends only on the arguments being expressible as functions of the density. Any variables which satisfy this criterion are technically usable.

The general scheme now is to investigate these fundamental variables, or combinations of these which may be of interest, on the basis that their largest values must occur within an interval prescribed by the change of density corresponding to a given shock. These largest values will therefore be found at points where the derivative with respect to density is zero or at the end points of the interval.

4. GENERAL RESULTS

The most convenient parameter in the present context to describe a family of waves is the maximum density change or, equivalently, ρ_m/ρ_0, where ρ_m is the maximum density. It would be possible to use wave speed, as was done in (KG), maximum pressure, or other parameters, but less convenient since these are nearly explicit functions of density but not conversely.